

MAT101 ANALİZ I 2. SÜREÇ SORU ÇÖZÜMLERİ

1) $\{(x,y) \in \mathbb{R}^2 : |y| = \lfloor x \rfloor\}$ grafiğini çiziniz.

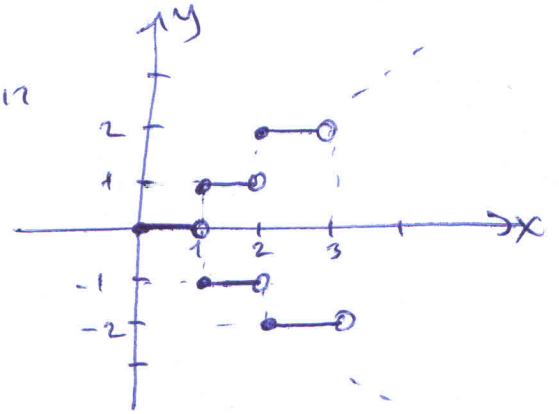
$$-2 \leq x < -1 \Rightarrow \lfloor x \rfloor = -2 \text{ olup } |y| = 2 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{tanımsız}$$

$$-1 \leq x < 0 \Rightarrow \lfloor x \rfloor = -1 \Rightarrow \text{diğer } |y| = 1$$

$$0 \leq x < 1 \Rightarrow \lfloor x \rfloor = 0 \Rightarrow |y| < 0 \Rightarrow \underline{y = 0}$$

$$1 \leq x < 2 \Rightarrow \lfloor x \rfloor = 1 \Rightarrow |y| = 1 \Rightarrow y = \pm 1$$

$$2 \leq x < 3 \Rightarrow \lfloor x \rfloor = 2 \Rightarrow |y| = 2 \Rightarrow y = \pm 2$$



2) a) $f_1(x) = \sqrt{9-x^2}$, $f_2(x) = \log\left(\frac{x+1}{x+2}\right)$, $f = f_1 + f_2$, $D_f = D_{f_1} \cap D_{f_2}$

$$D_{f_1} = \{x \in \mathbb{R} : 9-x^2 \geq 0\} = [-3, 3], \quad D_{f_2} = \{x \in \mathbb{R} : \frac{x+1}{x+2} > 0\} = (-\infty, -1) \cup (2, +\infty)$$

$$D_f = [-3, 3] \cap ((-\infty, -1) \cup (2, +\infty)) = [-3, -1) \cup (2, 3]$$

$$b) D_f = \{x \in \mathbb{R} \mid 0 \leq 2 \arccos x < \frac{\pi}{2}, \frac{\pi}{2} < \arccos x \leq \pi\}$$

$$= [-1, 1] \setminus \left\{ \pm \frac{1}{\sqrt{2}} \right\}$$

3) $a: \mathbb{N} \rightarrow \mathbb{R}$ her fonksiyona \mathbb{R} de bir dizi denir.
n tane a_n

$(a_n) \subset \mathbb{R}$ sınırlı dizi $\Leftrightarrow \exists M > 0 : |a_n| \leq M \quad \forall n \in \mathbb{N}$.

$$\forall n \in \mathbb{N} \text{ için } |a_n| = \left| \frac{(-1)^n \cdot n + 10}{\sqrt{n^2 + 1}} \right| \leq \frac{|(-1)^n \cdot n| + 10}{\sqrt{n^2 + 1}} \leq \frac{n + 10}{\sqrt{n^2 + 1}}$$

$$n < \sqrt{n^2 + 1} \text{ olduğundan } \frac{1}{n} > \frac{1}{\sqrt{n^2 + 1}} \text{ dir. } \Rightarrow$$

$$|a_n| \leq \frac{n+10}{\sqrt{n^2+1}} \leq \frac{n+10}{n} = 1 + \frac{10}{n} \leq 11 = M.$$

$$4) a) \arccos\left(-\frac{\sqrt{3}}{2}\right) + \arctan(-1) = \pi - \arccos\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\pi}{4}\right) = \pi - \frac{\pi}{6} - \frac{\pi}{4} \\ = \frac{5\pi}{6} - \frac{\pi}{4} = \frac{20 - 6\pi}{24} = \frac{14\pi}{24} = \frac{7\pi}{12}$$

$$b) \cos\left(\pi \left(\sinh(\ln 2)\right)\right) = \cos\left(\pi \cdot \frac{e^{\ln 2} - e^{-\ln 2}}{2}\right) = \cos\left(\pi \cdot \frac{(2 - 1/2)}{2}\right) = \cos\left(\frac{3\pi}{4}\right) \\ = -\frac{\sqrt{2}}{2}$$

$$c) f(-1) = \frac{1}{2^{-1}} + 1 = 2 + 1 = 3, \quad f(0) = \tan 0 = 0, \quad f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$f(2) = \frac{2}{2^2+1} = \frac{2}{4+1} = \frac{2}{5}$$